ALG III 3/19/18
Natural Logarithm and Base e

$A = P(1+\frac{r}{n})^{nt}$

WarmUp: You invest $1 at a 100% interest rate for one year. You can't remember how often the interest is compounded. You are curious as to whether there is a *maximum* amount of money you could have at the end of the year.

Complete the table.

Interest is Compounded	n	$1(1+\frac{1}{n})^n$
annually	1	2
semiannually	2	2.25
quarterly		
bimonthly		
monthly		
weekly		
daily		
hourly		
each minute		
each second		

e

So as n gets super huge, the value of $\left(1+\frac{1}{n}\right)^n$ gets closer and closer, but never exceeds, e.

Let's see what impact this has on our Compound Interest equation, and answers our original question.

$$A = P\left(1+\frac{r}{n}\right)^{n\cdot t}$$

$$A = P\left(1+\frac{1}{\left(\frac{n}{r}\right)}\right)^{\left(\frac{n}{r}\right)r\cdot t}$$ Pause here and make sure you believe this is equivalent...

$\frac{1}{\frac{n}{r}} = \frac{1}{n\cdot\frac{1}{r}} = \frac{r}{n}$

$$A = P\left(\left(1+\frac{1}{\left(\frac{n}{r}\right)}\right)^{\frac{n}{r}}\right)^{r\cdot t}$$ Again, pause... same thing?

Now if n gets super huge, $\left(1+\frac{1}{\left(\frac{n}{r}\right)}\right)^{\frac{n}{r}}$ approaches e, and our formula becomes $A = P\cdot e^{r\cdot t}$

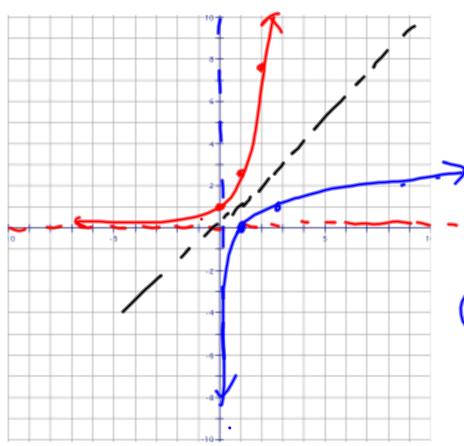
Ex#1: You invest $200 in an account that pays 4.2% annual interest, but you don't know how often it is compounded. What is the most money you could possibly have at the end of 5 years?

$$A = 200(e)^{0.042 \cdot 5} = 246.74$$

Ex#2: Given $f(x) = e^x$ $\quad y = e^x = 1(e)^x \approx 1(2.718)^x$

a) Complete the table and graph $y = f(x)$

x	$f(x)$
-1	
0	
1	
2	



$y = e^x$

$x = e^y$

$\log_e x = \log_e e^y$

$\log_e x = y$

$\ln x = y$

b) On the same axes, graph the inverse. Find the equation of the inverse.

a) $\ln e^8$

$\log_e e^8$

8

b) $5\log_e e^4$

$5(4)$

20

$5\ln e^4$

$\ln (e^4)^5$

$\ln e^{20}$

20

c) $e^{2\ln 3}$

$e^{2\ln 3}$

$e^{\ln 3^2}$

3^2

9

Ex#4: Solve each equation.

a) $\frac{2e^x}{2} = \frac{38}{2}$

$e^x = 19$

$\ln e^x = \ln 19$

$x = 2.94$

b) $\ln e^{x+8} = 12$

$x+8=12$

$x=4$

c) $2e^{x-4} + 8 = 12$

$2e^{x-4} = 4$

$e^{x-4} = 2$

$\ln e^{x-4} = \ln 2$

$x - 4 = \ln 2$

$x = \ln 2 + 4$

$x \approx 4.69$

d) $5\log_e(x-4) + 3 = 38$

$5\ln(x-4) + 3 = 38$

$5\ln(x-4) = 35$

$e^{\ln(x-4)} = e^7$

$x - 4 = e^7$

$x = e^7 + 4$

$x = 1100.63$